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# New integrable cases in the problem of a rigid body motion in ideal incompressible fluid

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*The paper considers the classical problem of motion of a rigid body in a multivariable ideal incompressible fluid — Kirchhoff problem. The study shows that in the case when the parameters of the Hamiltonian matrix are reduced to a diagonal form, the elements of diagonal matrices have no restrictions, namely, all the 9 parameters are independent and can take any value. It is shown that by using canonical transformations, equations of motion in an axisymmetric spherical force field are reduced to the form of Kirchhoff's equations describing the motion of a rigid body in a multivariable ideal incompressible fluid. It is stressed that the equations of the problem are integrated in quadratures with an arbitrary tensor of inertia, arbitrary location of the masses center and arbitrary quadratic part of the potential. Classical integrable Lagrange, Kovalevskaya, Goryachev – Chaplygin's cases are included in the found solution as a partial result.*

**Keywords:** Kirchhoff problem, axisymmetric spherical force field, spherical motion of a rigid body, Poincare theorem.

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