# Optimal bielliptic transition between coplanar elliptical orbits 

© S.A. Zaborskiy ${ }^{1}$, E.V. Kiriluk ${ }^{2}$<br>${ }^{1}$ S.P. Korolev Rocket and Space Public Corporation Energia, Korolev town, Moscow region, 141070, Russia<br>${ }^{2}$ Bauman Moscow State Technical University, 105055, Russia

An analytic solution of the problem of the bielliptic three-pulse transition between circular orbits and an analytic solution for the problem of the bielliptic transition between coaxial elliptic orbits are presented. These solutions were published in a number of sources. In this article, an attempt is made to generalize the existing results for the case of the transition between two specified points belonging to boundary disparate elliptic orbits when the radial values of the transition orbit apogee are specified. An analysis of the obtained relationship for the size of the total pulse increment of the velocity necessary for the performing the bielliptic maneuver is given. The limiting case in which the bielliptic transition degenerates into a biparabolic transition is considered. The dependences of the transition orbit parameters and the conditions under which the three-pulse transition can have advantages over the two-pulse one are established.

Keywords: optimal maneuver, coplanar transition, interorbital transition, biparabolic maneuver, bi-elliptic maneuver, analytical solution, impulse maneuver

## REFERENCES

[1] Hohmann W. Die Erreichbrakeit der Himmelskörper. Oldenbourg, 1925, 88 S.
[2] Gobetz F.W., Doll J.R. AIAA Journal, 1969, vol. 7, no. 5, pp. 801-834. DOI: 10.2514/3.5231
[3] Battin R.H. An Introduction to the Mathematics and Methods of Astrodynamics. New York, AIAA, 1999, 796 p.
[4] Lawden D.F. Optimal Trajectories for Space Navigation. London, Batterworths, 1963, 123 p .
[5] Horner J.M. ARS Journal, 1962, vol. 32, no. 1, pp. 95-96.
[6] Horner J.M. AIAA Journal, 1963, vol. 1, no. 7, pp. 1707-1708. DOI:10.2514/3.1906
[7] Hoelker R.F., Silber R. The Bi-Elliptical Coplanar Circular Orbits. Proceeding of the 4th Symposium on Ballistic Missiles and Space Technology, New York, Pergamon, 1961, vol. 3, pp. 164-175.
[8] Marchal C. Astronautica Acta, 1965, vol. 11, no. 6, pp. 432-445.
[9] Zaborsky S. Journal of Guidance, Control, and Dynamics, 2014, vol. 37, no. 3, pp. 996-1000. Available at: http://arc.aiaa.org/doi/pdf/10.2514/1.62072 (accessed September 20, 2016).
[10] Gill P.E., Murray W.M., Saunders M.A., Wright M.H. User's Guide for NPSOL (version 4.0). A Fortran Package for Nonlinear Programming. Technical Report SOL 86-2. Department of Operations Research, Stanford University Publ., 1986, 44 p.

Zaborskiy S.A., Cand. Sc. (Eng.), Head of the Launch Vehicle Sector, Department of Space Ballistics, S.P. Korolev Rocket and Space Public Corporation Energia, Assistant

Lecturer, Department of Aerophysical Mechanics and Motion Control, Moscow Institute of Physics and Technology. Author of 4 research publications.
e-mail: Sergey.Zaborsky@rsce.ru
Kiriluk E.V., Postgraduate student, Assistant Lecturer, Department of Space Flight Dynamics and Control, Bauman Moscow State Technical University, engineer, S.P. Korolev Rocket and Space Public Corporation Energia. Author of 8 research publications in the field of optimal control of launch vehicles, flight dynamics of flight vehicles.
e-mail: Elena.Kirilyuk@rsce.ru

